Math 327: Discrete Mathematics **(4-0-4)**
November 8, 2017

Catalog Description: Introduction to mathematical abstraction. Topics include sets, set operations, functions, relations, sequences, series, recurrence relations, mathematical induction, equivalence relations, elementary number theory, graph theory.

Prerequisite: MATH252, or junior standing and MATH111, both with grade C or better.

Course Objectives: After completing this course, students will be able to:

1. Construct statements about sets.
2. Establish and use properties of relations.
3. Apply number theory to perform computations and construct proofs about integers.
4. Perform computations with sequences and series.
5. Apply concepts from graph theory to solve problems.

Learning Outcomes and Performance Criteria

1. Construct statements about sets.

   Core Criteria:

   (a) Give the definition of a set using interval notation, using listing and set-builder notation.
   (b) Use De Morgan's laws to make statements about sets.
   (c) Construct the complement of a set.
   (d) Show that a set is a subset.
   (e) Find the cardinality of a finite set.
   (f) Identify infinite sets that are countable and uncountable.
   (g) Construct the power set of a given set.
   (h) Construct the Cartesian product of two sets.

   Additional Criteria:

   (a) Construct and interpret Venn diagrams.
   (b) Prove closure of sets under various operations.
   (c) Prove that two sets are equal by showing that each set is the subset of the other.

2. Establish and use properties of relations.

   Core Criteria:

   (a) Give examples of relations that are reflexive, symmetric, transitive and anti-symmetric.
   (b) Identify if a given relation is reflexive, symmetric, transitive and/or anti-symmetric.
   (c) Identify if a relation is an equivalence relation.
   (d) Identify equivalence classes for a given equivalence relation.
   (e) Determine if an element is in an equivalence class or not.

(f) Give the partition of a set based on an equivalence relation.

(g) Decide if a given relation is a function. Determine its domain and range.

(h) Determine and prove whether a function is injective, surjective and/or bijective.

(i) Use functions to establish the cardinality of a set.

(j) Form new functions by using composition of functions. Determine the domain and range of the composition.

Additional Criteria:

(a) Prove that a relation is a partial order.

(b) Construct a Hasse diagram for partial order.

(c) Identify maximal and minimal elements of a partially ordered set.

3. Apply number theory to perform computations and construct proofs about integers.

Core Criteria:

(a) Compute the greatest common divisor (gcd) and least common multiple (lcm) for a pair of integers.

(b) Use the Euclidean algorithm to compute the gcd of a pair of numbers.

(c) Apply the division algorithm. Be able to write a number in the form of $a = qb + r$ for given a and b.

(d) Prove statements involving divisibility of integers.

(e) Perform modular arithmetic.

(f) Solve linear congruences.

Additional Criteria:

(a) Prove that $\sqrt{n}$ is irrational for n not a perfect square.

(b) Apply number theory to RSA encryption.

(c) Solve a problem using Chinese Remainder Theorem.

4. Perform computations with sequences and series.

Core Criteria:

(a) Construct a sequence recursively.

(b) Determine an explicit closed form expression for a given sequence.

(c) Identify arithmetic and geometric sequences.

(d) Find closed form expressions for finite series.

(e) Use induction to prove statements about sequences and series.

(f) Solve first and second order linear difference equations.

Additional Criteria:

(a) Find the sum of a convergent geometric series.

5. Apply concepts from graph theory to solve problems.

Core Criteria:

(a) Define a graph and a directed graph formally (vertices, edges).

(b) Show two graphs are isomorphic by defining a graph isomorphism.

(c) Recognize a bipartite graph and a complete graph ($\mathcal{K}_n$ or $\mathcal{K}_{n,m}$).

(d) Show a graph is a subgraph of another graph by defining a graph injection.

(e) Use Euler's Formula ($V - E + F = 2$)

(f) Apply Kuratowski's theorem to show a graph is not planar.

(g) Find an Eulerian circuit in a given graph or show that one does not exist. (e.g. The Bridges of Königsberg)

(h) Find a Hamiltonian cycle in a given graph or show that one does not exist.

(i) Define the adjacency matrix for a graph.

(j) Use the adjacency matrix to find the shortest path between given vertices.

Additional Criteria:

(a) Colorings (Four Color Map Theorem)

(b) Trees