**MATH 342 :** Linear Algebra II **(3-0-3)** 10/17/18

**Catalog Description:** A continuation of the topics of MATH 341 to the setting of abstract vector spaces. Includes the study of orthogonality, inner product spaces, eigenvalues and eigenvectors, matrix decompositions and a more advanced study of linear transformations.

**Course Objectives:**

1. Use orthogonality in $\mathbb{R}^n$.
2. Understand vector spaces.
3. Understand and work with the inner product.
4. Apply optimization algorithms to solve linear and non-linear problems.

**Learning Outcomes and Performance Criteria**

1. Understand the significance of orthogonality in $\mathbb{R}^n$.

   Core Criteria:

   (a) Find the representation of a vector with respect to an orthogonal or orthonormal basis by projection.

   (b) Determine whether a set of vectors is orthogonal or orthonormal.

   (c) Given a basis for a subspace of $\mathbb{R}^n$, find a basis for the orthogonal complement of the space.

   (d) Given a basis for a subspace $W$ of $\mathbb{R}^n$ and a $\mathbf{v}$ in $\mathbb{R}^n$, find the unique orthogonal decomposition $\mathbf{v} = \mathbf{w} + \mathbf{w}^{\perp}$ with $\mathbf{w}$ in $W$ and $\mathbf{w}^{\perp}$ in $W^{\perp}$.

   (e) Apply the Gram-Schmidt process to a set of vectors; find the $QR$ factorization of a matrix.

   (f) Give the spectral decomposition of a symmetric matrix.

2. Understand and work with general vector spaces.

   Core Criteria:

   (a) Given a set of objects and definitions of addition and scalar multiplication,

      i. prove any of the properties of a vector space that hold,

      ii. give specific counterexamples to any properties that do not hold,

      iii. if there is a zero vector, determine what it is; if all vectors have inverses, give the general form of the inverse of a vector,

      iv. determine whether the set with those operations is a vector space.

   (b) Determine whether a subset of a vector space is a subspace.

   (c) Determine whether a set of vectors is linearly independent; if not, give one as a linear combination of the others.

   (d) Determine whether a set of vectors is a basis for a vector space; if not, tell why.

   (e) Give the representation of a vector with respect to a given basis.

(f) Extend a set to a basis for a given space.

(g) Reduce a spanning set to a basis.

(h) Given the representation of a vector with respect to one basis, determine its representation with respect to another.

(i) Determine the change-of-basis matrix from one basis to another.

(j) Determine whether a given transformation is linear.

(k) Given the action of a linear transformation on basis vectors, find the linear transformation of any vector.

(l) Determine whether a given vector is in the kernel or range of a linear transformation. Describe the kernel and range of a linear transformation.

(m) Determine whether a given linear transformation is (a) one-to-one, (b) onto.

(n) Determine whether two vector spaces are isomorphic. If they are, give an isomorphism from one to the other.

(o) Determine the matrix of a linear transformation with respect to given bases.

3. Understand and work with the inner product.

   Core Criteria:

   (a) Determine whether a given operation on a vector space is an inner product.

   (b) Compute the inner product of two vectors, norm of a vector, distance between two vectors. Determine whether two vectors are orthogonal.

   (c) Apply the Gram-Schmidt process to a set of orthogonal vectors to obtain an orthogonal basis.

   (d) Compute the least squares line or parabola for a set of data points. Compute the least squares solution to a system of equations. Solve problems involving least squares approximation.

   (e) Find the standard matrix of the approximation onto a subspace; find the projection of a vector onto a subspace.

   (f) Find the singular values of a matrix; find the singular value decomposition of a matrix.

4. Apply optimization algorithms to solve linear and non-linear problems.

   Core Criteria:

   (a) Find the maximum and/or minimum value of a quadratic form $Q(\mathbf{x})$, where $\mathbf{x}$ ranges over a set of vectors that satisfy some constraint using eigenvalues and eigenvectors.

   (b) Use matrix decomposition (such as LU, QR, SVD, Cholesky decomposition) to solve optimization problems.

   (c) Use Gradient descent methods to solve optimization problems.

   (d) Be able to check conditions under which different optimization techniques are appropriate for a given problem.