Math 310: Mathematical Structures **(4-0-4)**
November 10, 2016

Catalog Description: Introduction to proof and mathematical abstraction. Topics include logical statements, sets, set operations, functions, and relations.

Prerequisite: MATH252 with grade C or better.

Course Objectives: After completing this course, students will be able to:

1. Construct proofs.
2. Prove statements about sets.
3. Establish and use properties of relations.

Learning Outcomes and Performance Criteria

1. Construct proofs.

   Core Criteria:

   (a) Determine whether an object satisfies a definition. If not, explain why not.
   (b) Give an example that satisfies a given definition.
   (c) Prove a given statement is true or provide a counter example.
   (d) Compute the greatest common divisor (gcd) and least common multiple (lcm) for a pair of integers.
   (e) Prove statements involving divisibility of integers.
   (f) Compute $a \mod n$ for a given a and n.
   (g) Construct the negation, contra-positive and the converse to a given statement. Give the negation of a quantifier.
   (h) Construct a direct proof, proof by contradiction and proof by contrapositive.
   (i) Construct a proof by induction.
   (j) Use the Well Ordering Principle in proofs.
   (k) Use induction to prove statements about sequences and series.

2. Prove statements about sets.

   Core Criteria:

   (a) Give the definition of a set using interval notation, listing and set-builder notation.
   (b) Use De Morgan's laws to prove statements about sets.
   (c) Construct the complement of a set. Prove statements that involve the complement of a set.
   (d) Prove that a set is a subset.
   (e) Prove that two sets are equal by showing that each set is the subset of the other.
   (f) Identify infinite sets that are countable and uncountable.
   (g) Prove closure of sets under various operations.

   Additional Criteria:

   (a) Construct and interpret Venn diagrams.

(b) Construct the Cartesian product of two sets. Prove statements that involve the Cartesian product of two sets.

3. Establish and use properties of relations.

   Core Criteria:

   (a) Give examples of relations that are reflexive, symmetric, transitive and anti-symmetric.
   (b) Prove that a given relation is reflexive, symmetric, transitive and/or anti-symmetric.
   (c) Prove that a relation is an equivalence relation.
   (d) Identify equivalence classes for a given equivalence relation.
   (e) Determine if an element is in an equivalence class or not.
   (f) Give the partition of a set based on an equivalence relation.
   (g) Decide if a given relation is a function. Determine its domain and range.
   (h) Determine and prove whether a function is injective, surjective and/or bijective.
   (i) Use functions to establish the cardinality of a set.
   (j) Find an image or inverse image of a set under a function.
   (k) Find the image and pre-image of the union and intersection of sets.
   (l) Give proofs or counterexamples of statements about images or inverse images of sets under functions.
   (m) Form new functions by using composition of functions. Determine the domain and range of the composition.
   (n) Prove whether compositions of injective/surjective/bijective functions are injective/surjective/bijective.

   Additional Criteria:

   (a) Prove that a relation is a partial order.
   (b) Construct a Hasse diagram for partial order.
   (c) Identify maximal and minimal elements of a partially ordered set.